Problem Q No - Expand $\frac{1}{z(z^2-3z+2)}$ for the regi...

~~(i) $0<|z|<1$,~~

(i) $0<|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$.

Solu^n. (i) Here, $f(z) = \frac{1}{z(z^2-3z+2)}$

$$= \frac{1}{z(z-1)(z-2)}$$

$$= \frac{1}{2z} - \frac{1}{z-1} + \frac{1}{2(z-2)}$$

$$= \frac{1}{2z} + \frac{1}{z-1} - \frac{1}{4\left(1-\frac{z}{2}\right)}$$

$$= \frac{1}{2z} + (1-z)^{-1} - \frac{1}{4}\left(1-\frac{z}{2}\right)^{-1}$$

$$= \frac{1}{2z} + (1+z+z^2+z^3+\cdots) - \frac{1}{4}\left(1+\frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right)$$

(ii) Here, $f(z) = \frac{1}{z(z^2-3z+2)}$

$$= \frac{1}{2z} - \frac{1}{z-1} + \frac{1}{2(z-2)}$$

$$= \frac{1}{2z} - \frac{1}{z\left(1-\frac{1}{z}\right)} + \left(\frac{-1}{4\left(1-\frac{z}{2}\right)}\right)$$

$$= \frac{1}{2z} - \frac{1}{z}\left(1-\frac{1}{z}\right)^{-1} - \frac{1}{4}\left(1-\frac{z}{2}\right)^{-1}$$

$$= \frac{1}{2z} - \frac{1}{z}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right) - \frac{1}{4}\left(1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right)$$

(iii) Here, $f(z) = \dfrac{1}{z(z^2 - 3z + 2)}$

$$= \frac{1}{2z} - \frac{1}{z-1} + \frac{1}{2(z-2)}$$

$$= \frac{1}{2z} - \frac{1}{z\left(1-\frac{1}{z}\right)} + \frac{1}{2z\left(1-\frac{2}{z}\right)}$$

$$= \frac{1}{2z} - \frac{1}{z}\left(1-\frac{1}{z}\right)^{-1} + \frac{1}{2z}\left(1-\frac{2}{z}\right)^{-1}$$

$$= \frac{1}{2z} - \frac{1}{z}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right) + \frac{1}{2z}\left(1 + \frac{2}{z} + \frac{2^2}{z^2} + \frac{2^3}{z^3} + \cdots\right)$$

Q.No→ Expand $f(z) = \dfrac{1}{(z+1)(z+3)}$ in a Laurent's series valid for the regions

(i) $|z| < 1$ (ii) $1 < |z| < 3$ (iii) $|z| > 3$ (iv) $1 < |z+1| < 2$.

Solⁿ:- Here, $f(z) = \dfrac{1}{(z+1)(z+3)}$

$$= \frac{1}{2(z+1)} - \frac{1}{2(z+3)}$$

(i) For $|z| < 1$, we have

$$f(z) = \frac{1}{2(z+1)} - \frac{1}{2(z+3)}$$

$$= \frac{1}{2}(1+z)^{-1} - \frac{1}{2\cdot 3\left(1+\frac{z}{3}\right)}$$

$$= \frac{1}{2}(1+z)^{-1} - \frac{1}{6}\left(1+\frac{z}{3}\right)^{-1}$$

$$= \frac{1}{2}\left\{1 - z + z^2 - z^3 + \cdots\right\} - \frac{1}{6}\left\{1 - \frac{z}{3} + \frac{z^2}{3^2} - \frac{z^3}{3} + \cdots\right\}$$

(ii) For, $1 < |z| < 3$, we have

$$f(z) = \frac{1}{2(z+1)} - \frac{1}{2(z+3)}$$

$$= \frac{1}{2}\left(\frac{1}{z\left(1+\frac{1}{z}\right)}\right) - \frac{1}{2\cdot 3\left(1+\frac{z}{3}\right)}$$

$$= \frac{1}{2}\left\{\frac{1}{z}\left(1+\frac{1}{z}\right)^{-1} - \frac{1}{6}\left(1+\frac{z}{3}\right)^{-1}\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{z}\left(1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots\right) - \frac{1}{6}\left(1 - \frac{z}{3} + \frac{z^2}{3^2} - \frac{z^3}{3^3} + \cdots\right)\right.$$

(iii) For, $|z| > 3$, we have

$$f(z) = \frac{1}{2}\left(\frac{1}{z\left(1+\frac{1}{z}\right)}\right) - \frac{1}{2\cdot z\left(1+\frac{3}{z}\right)}$$

$$= \frac{1}{2}\left\{\frac{1}{z}\left(1+\frac{1}{z}\right)^{-1} - \frac{1}{z}\left(1+\frac{3}{z}\right)^{-1}\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{z}\left(1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots\right) - \frac{1}{z}\left(1 - \frac{3}{z}\right.\right.$$

$$+\frac{3^2}{z^2}-\frac{3^3}{z^3}+\cdots\Big\}$$

(iv) Let $z+1=u$ ∴ $0<|z+1|<2 \Rightarrow 0<|u|<2$ in this case, $f(z)=\frac{1}{(z+1)(z+3)}$

$$=\frac{1}{u(u+2)}=\frac{1}{2u\left(1+\frac{u}{2}\right)}$$

$$=\frac{1}{2u}\left(1+\frac{u}{2}\right)^{-1}$$

$$=\frac{1}{2u}\left[1-\frac{u}{2}+\frac{u^2}{2^2}-\frac{u^3}{2^3}+\cdots\right]$$

Q No → Expand $f(z)=\frac{z+3}{z(z^2-z-2)}$ in Powers of z

(i) within the unit circle about the origin

(ii) within the annulus region between the concentric circles about the origin having radii 1 and 2 respectively.

(iii) the exterior to the circle with centre as origin and radius 2 i.e. for $|z|>2$.

Soluⁿ (i) We have $f(z)=\frac{z+3}{z(z^2-z-2)}$

$$=\frac{z+3}{z(z+1)(z-2)}=\frac{-3}{2z}+\frac{2}{3(z+1)}+\frac{5}{6(z-2)}$$

$$=-\frac{3}{2z}+\frac{2}{3}(1+z)^{-1}-\frac{5}{12}\left(1-\frac{z}{2}\right)^{-1}$$

$$= -\frac{3}{2z} + \frac{2}{3}\left(1 - z + z^2 - z^3 + \cdots\right) - \frac{5}{12}\left(1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{3} + \cdots\right)$$

(ii) $$f(z) = \frac{z+3}{z(z^2-z-2)} = \frac{z+3}{z(z+1)(z-2)}$$

$$= \frac{-3}{2z} + \frac{2}{3(z+1)} + \frac{5}{6(z-2)}$$

$$= -\frac{3}{2z} + \frac{2}{3z}\left(1+\frac{1}{z}\right)^{-1} - \frac{5}{12}\left(1-\frac{z}{2}\right)^{-1}$$

$$= -\frac{3}{2z} + \frac{2}{3z}\left(1 + \frac{1}{z} - \frac{1}{z^2} + \cdots\right) - \frac{5}{12}\left(1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right)$$

(iii) $$f(z) = \frac{z+3}{z(z^2-z-2)} = \frac{z+3}{z(z+1)(z-2)}$$

$$= \frac{-3}{2z} + \frac{2}{3(z+1)} + \frac{5}{6(z-2)} = -\frac{3}{2z} + \frac{2}{3z}\left(1+\frac{1}{z}\right)^{-1} - \frac{5}{6z}\left(1-\frac{2}{z}\right)^{-1}$$

$$= -\frac{3}{2z} + \frac{2}{3z}\left(1 + \frac{1}{z} - \frac{1}{z^2} + \cdots\right) - \frac{5}{6z}\left(1 - \frac{2}{z} + \frac{2^2}{z^2} + \frac{2^3}{z^3} + \cdots\right).$$